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PULSE FREQUENCY MODULATION

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GODDARD SPACE FLIGHT CENTER
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ABSTRACT

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This paper is concerned with the heuristic development of the basic features of pulse-frequency modulation, an information encoding technique which has found use in a number of spacecraft. The primary advantages have been its noise-immunity characteristics combined with its ease of generation. A description of the present method of formatting and synchronization is presented to illustrate the convenient handling of both analog and digital data. In the detection process for PFM signals, a set of contiguous unmatched filters is used to enhance the signal-to-noise ratio. To examine the effects of Rayleigh noise on the output of these filters, the word-error probability is derived as a function of the energy per bit, the noise power density, and the degree of coding. The same development is given for the matched-filter case. Experimental results from spectral analyses of satellite recordings perturbed with random noise are presented to illustrate the noise-immunity characteristic of pulse-frequency modulation.

AUTHOR

PULSE-FREQUENCY MODULATION TELEMETRY

by
Robert W. Rochelle
Goddard Space Flight Center

I. INTRODUCTION

The problem of communicating over large distances with minimum power is particularly challenging in light of the advances in information theory during the last two decades. With this theory as a guide, a number of interesting types of communication systems can be postulated which, in the limit, approach the theoretical maximum communication efficiency. The encoding methods of some of these communication systems are exceedingly complex in their implementation. The ideal type of system is one which retains the high communication efficiency with little loss in simplicity. Pulse-frequency modulation is an attempt to fulfill these two basic criteria.

Pulse-frequency modulation (PFM) has been successfully employed as the information encoding technique in a number of small (< 200 lbs.) scientific satellites and space probes where reduction of spacecraft power and weight are prime considerations. The use of such systems results in greater percentage of the spacecraft available for scientific instrumentation.

A. The Encoding Problem

The concept of modern telemetry includes the gathering and encoding of information at a remote station, transmitting this encoded signal to the receiving station, and there decoding the signal and presenting the measurements in an acceptable form. Attenuation of the transmitted signal is usually a salient factor because of the physical separation between the transmitting and receiving apparatus. If attenuation is low, the choices of the encoding and modulation methods are not critical; thus methods leading to simple and reliable systems are usually employed. With the introduction of attenuation in the transmission path, interference in various forms may perturb the signals and cause errors in the transmission. Proper encoding of the information, however, can minimize the effects of these "noise" perturbations. The amount of improvement or immunization to noise that can be accomplished is governed by Shannon's channel capacity theorem:[1]

$$C = B \log_2 \left(\frac{P + N}{N} \right) ,$$

where C = channel capacity (bits per second), B = channel bandwidth, P = signal power, and N = average noise power.

As the signal power is decreased, the bandwidth of the encoded signal must be increased to maintain the same channel capacity and probability of error.

Various schemes can be concocted to take advantage of Shannon's channel capacity theorem and thereby reduce the transmitter power for the same information rate. Kotel'nikov[2] has described the general theory of high-efficiency encoding. The problem is, of course, to approach the Shannon channel-capacity limit as closely as possible, without requiring unduly complex equipment.

A practical example of this problem is the telemetering of scientific data from satellites and space probes. Since the spacecraft power is limited, savings in transmitter power due to efficient coding can extend either the range, the information rate, or both. This improvement is greater in the small satellite class (<200 lbs.), where a larger percentage of the satellite weight is devoted to the transmitter power system.

B. History of PFM

With the entry of the United States into space exploration, a lightweight low-power telemetry system was needed. Such a system,[3] utilizing the noise-reducing properties of frequency modulation and the error-reducing properties of pulse-code modulation, was used in the first Vanguard scientific satellite and ultimately achieved orbit as Vanguard III in September 1959. The 48-channel telemetry encoder weighed six ounces (unpotted) and required 12 milliwatts. Pulse-frequency modulation, as the system was called, was used again on the Ionosphere Direct Measurements Satellite, Explorer VIII, orbited in November, 1960. In April 1961 the space probe Explorer X, measuring the interplanetary magnetic field, was sent to an altitude of 150,000 miles with pulse-frequency modulation as the encoding technique. Since then, four more scientific satellites have been orbited with pulse-frequency modulation telemeters: Explorer XII provided continuous measurements of the energetic particles in the Van Allen radiation belts out to 50,000 miles; the ionosphere satellite Ariel I was a joint cooperative effort between the United States and the United Kingdom, the experiments being supplied by the United Kingdom and the spacecraft by the United States; Explorer XIV was a follow-on for Explorer XII; and Explorer XV, launched in October 1962, performed a study of the artificially enhanced radiation belts. Two additional satellites to be launched in the near future — the Interplanetary Monitoring Probe and the follow-on to Ariel I — will both use pulse-frequency modulation as the encoding technique.

II. CHARACTERISTICS OF PULSE-FREQUENCY MODULATION

A. General Description.

Consider a time function $f_1(t)$ which is band-limited between zero and $1/2T_0$ cps. The function may be completely described by a series of impulses with separation T_0 and of area $f_1(nT_0)$ with $n = -\infty \cdots -2, -1, 0, 1, 2, \cdots +\infty$. As to the ways in which a sequence of these time samples may be encoded for transmission over a telemetry link, several choices are available. The method under consideration here is to encode the magnitude of the area of each impulse as the frequency of a pulsed subcarrier, the duration of the pulse being some fraction of the sample time T_0. Thus, a train of sequential pulses, each having a frequency proportional to the amplitude of a data sample, comprises the basic configuration for pulse-frequency modulation.

1. Format

The signals $f_m(t)$ may not necessarily be analog in nature. A sizable fraction of the experiments aboard spacecraft yield digital signals; and from a signal-to-noise viewpoint it is desirable to retain this digital character. The binary digits of the signal are combined and presented as a single digit of a higher-order base. The present state of the art of PFM telemetry allows the encoding of three binary bits as one digit to the base eight; that is, three binary digits are encoded as one of eight frequencies. A special digital pulsed-subcarrier oscillator[4] has been developed for this purpose; it is restricted to oscillate at only one of eight possible frequencies, depending on the value of its three-bit input. In this manner, the complexity of the switching circuitry in the encoder is materially reduced, since the readout of accumulators and scalers is accomplished three bits at a time instead of serially.

Since both analog and digital signals are encoded as pulse frequencies, they may be mixed in any order in the multiplexing. Any one channel may be subcommutated to extend the number of signals $f_m(t)$ which may be encoded, with a corresponding reduction in the signal bandwidth.

The presently used format for PFM specifies sixteen sequential channels as comprising a telemetry frame. The first channel is devoted to synchronization; the remaining fifteen are distributed between the analog and digital data to be telemetered. Sixteen sequential frames comprise a telemetry sequence of 256 pulses (sixteen of them devoted to synchronization).

The frame format and the sequence format are shown in Figure 1.

2. Synchronization

Synchronization is assured in two ways. The first is to increase the energy in the synchronization pulse by increasing its length 50 percent. The second is to utilize a unique frequency outside the data band for the sync pulse. To provide a means for identifying the subcommutated data, the frequency of the sync pulse in every other telemetry frame is stepped in sequence through eight frequencies. These frequencies lie in the data band and are the same as the eight frequencies of a digital oscillator. In noisy conditions, the energy at the sync frequency may be stored in a memory device over a number of frames, improving the sync signal-to-noise ratio. This, of course, increases the acquisition time but is a necessary compromise in regions of poor signal-to-noise ratio.

B. Design Considerations.

Let us investigate a few factors which are necessary in a PFM telemetry system. Consider again the analog time function $f_1(t)$. Let the maximum value of $f_1(t)$ be defined as the full-scale value F_1 of the channel. Now let F_1 be divided into N equal parts. Then the magnitude F_1/N represents the precision of the system and is the smallest discernible change between two samples of $f_1(t)$. The value of N is determined by the requirements of the experiment being telemetered. For small scientific satellites, a precision factor 1/N of 0.01 or one percent of the full-scale value is normally considered adequate. For experiments such as the flux-gate magnetometer, which depend upon measuring the modulation of the magnetic field by the satellite's spin, precisions to 0.1 percent are desirable, although accuracy better than one percent is not necessary. For analog signals encoded with PFM, the precision is a function of the signal-to-noise ratio. At close range, the precision is an order of magnitude better than at maximum range. This feature is equivalent to a variable bit-rate.

Since the frequency of the pulse contains the signal information, a measurement precision of one percent, for example, requires that the detection system be able to discern 100 different frequencies. The total frequency band B is therefore divided into 100 equal parts. The separation between frequencies, Δf_0, becomes B/100. The power spectra of the pulses have zeros of power density on each side of the center frequency at multiples of the reciprocal of the pulse length T; and T is determined by the sampling rate necessary to telemeter the highest frequency of $f_1(t)$ at least better than at the Nyquist rate. The zeros of the power spectrum should then fall at multiples of B/100, or $\Delta f_0 = 1/T$. This gives the relation between the bandwidth, precision, and pulse length:

$$B = \frac{N}{T}, \tag{1}$$

where B = total bandwidth, 1/N = precision constant, and T = pulse length.

The foregoing analysis was made on the basis of the sampling time of a single channel $f_1(t)$ determining the pulse length T. Since time-division multiplexing is used, the pulse length must be shortened in proportion to the number of commutated channels. Thus the bandwidth B will increase in proportion to the number of commutated channels.

With the application of Equation 1 we are, in effect, predicting that our sample will assume any one of N or, in this case, 100 discrete amplitudes. The power spectrum for any one of these amplitudes is centered in one of the 100 equal parts of width Δf_0 in the band B.

A first-order solution to the detection problem is to arrange 100 contiguous bandpass filters with bandwidth $\Delta f_0 = 1/T$ to cover the bandwidth B. A maximum-likelihood detector on the outputs of the filters can select the filter with the greatest output. Thus the signal is quantized into one of 100 possible levels. The original amplitude of a sample would not necessarily cause the frequency to fall exactly in the center of any one of the 100 filters. The output then would diminish in that filter and rise in an adjacent filter. As the amplitude of the sample is changed, the maximum-likelihood detector will indicate that the adjacent filter has acquired the signal only when the response in the adjacent filter rises above the output of the original filter. In conditions of good signal-to-noise ratio the filter with the greatest output can be gated into a discriminator to measure the frequency more precisely than one percent.

Rather than sampling only at times nT, the pulse frequency can be a continuous representation of the amplitude of the sample for the whole duration of the pulse T. The use of a discriminator on the output of the contiguous filters can determine not only the average amplitude during the time T but also the rate of change of amplitude. Having the latter information is equivalent to doubling the sampling rate.[5]

III. NOISE ANALYSIS

Pulse-frequency modulation can be exemplified by a special set of binary coded sequences which are selected because of their ease of generation. Of primary interest are the characteristics of this code set in the presence of noise. Various methods are available to aid in recovering the signal set from noise; the optimum method is to use matched filters with maximum-likelihood detection. The number of filters employed is a function of the desired efficiency of encoding: to obtain n-bit precision for a single word, 2^{n-1} filters are required. The matched-filter technique has been well reported in the literature.[6,7] The idea is to force the power spectrum of the impulsive response of the filter to be identical to the power spectrum of the signal. Although the use of matched filters is the most desirable method, there are many situations in which unmatched filters can markedly reduce hardware and implementation complexity without too greatly degrading the signal-to-noise ratio. Noise analyses differ, depending on whether one considers the filter in the detection system to be matched or unmatched. We shall first concern ourselves with the unmatched filter in the detection system, and calculate the probability of word error as a function of signal-to-noise ratio.

A. Unmatched Filter.

The use of unmatched filters for PFM signals in which the number of quantized levels exceeds eight ($n = 3$) can simplify the detection system's operation as compared to that of a matched-filter set. The unmatched-filter set would consist of N contiguous bandpass filters where N is the number of quantized levels in the signal. White additive Gaussian noise with a power density of N_0 watts/cps, when passed through one of these filters, will have a variance of σ^2 watts in the output. We may represent the noise output of a narrow-band filter by the use of in-phase and quadrature components:

$$u(t) = x(t)\cos\omega_0 t + y(t)\sin\omega_0 t \,, \tag{2}$$

where $x(t)$ and $y(t)$ are random variables. The frequency spectra of $x(t)$ and $y(t)$ do not extend up to the center frequency ω_0. The mean-square-deviation or variance σ^2 is the sum of the variance of the in-phase and quadrature components, or $\sigma^2 = \sigma_I^2 + \sigma_Q^2$ (the covariance between the in-phase and quadrature components is zero); also, $\sigma^2 = \sigma_x^2/2 + \sigma_y^2/2$.

1. Ideal Filter

Consider a single ideal filter of bandwidth $\Delta\omega$ and center frequency ω_0 . Although the noise output of the filter cannot be predicted, certain probabilities may be assigned to the noise characteristics to permit evaluation of the long-term effect of the presence of the noise. For the unmatched filter case we shall concern ourselves mainly with the probability density function of the envelope amplitude of the noise output, since an uncorrelated detector produces a voltage proportional to the envelope of its input.

First, the amplitude of the noise in the output of the unmatched filter, like that of the input, has a Gaussian probability density function. Positive and negative amplitudes are equi-probable, with zero being the most probable value.

When the output of the filter is passed through a simple detector with the characteristics

$$\left.\begin{aligned} e_d &= -n(t) \text{ for } n(t) \le 0 , \\ e_d &= +n(t) \text{ for } n(t) \ge 0 , \end{aligned}\right\} \qquad (3)$$

where $n(t)$ = noise voltage input and e_d = detector voltage output, full-wave rectification of the output occurs. Smoothing of this rectified output yields $\sqrt{|x(t)|^2 + |y(t)|^2}$, which is the amplitude of the envelope of $u(t)$ (Equation 2).

The amplitudes of the noise output of the detector have a Rayleigh probability density function[8] given by:

$$P(r) = \frac{r}{\sigma_x^2} e^{-\frac{r^2}{2\sigma_x^2}}$$

where r is the amplitude of the envelope of $u(t)$ and σ_x^2 is the variance of the noise $x(t)$. Figure 2b includes a plot of a Rayleigh probability density function for noise alone.

The addition of a sinusoidal signal to the noise causes a shift in the envelope-amplitude probability density function as predicted by Rice[9] in his equation (3.10-11). Adding a sine-wave carrier of amplitude C_0 and phase θ_0 to the noise displaces the peak of the probability density function as shown in Figure 2(a). The probability of an envelope with amplitude r becomes:

$$p(r) = \frac{r}{\sigma_x^2} e^{-\frac{r^2 + C_0}{2\sigma_x^2}} I_0\left(\frac{r C_0}{\sigma_x}\right)$$

where

$$I_0\left(\frac{r C_0}{\sigma_x}\right) = \frac{1}{2\pi} \int_0^{2\pi} e^{\frac{r C_0 \cos\left(\theta - \theta_0\right)}{\sigma_x^2}} \, d\theta$$

is the modified Bessel function of the first kind of order zero.

2. Word Error Probability

We have examined the action of an individual ideal bandpass filter and, given a signal-with-noise input, determined the probability of a particular output. Let us now look at the action of signal and noise as an input to a bank of parallel bandpass filters (the band-edges of the filters are touching). With N filters a frequency range of $N\Delta\omega/2\pi$ is covered. It should be remembered that the primary advantage of PFM is that each PFM word is orthogonal to any other word in that PFM set. This forces a uniqueness in the power spectrum of each word, resulting in a bandwidth equal to the products of the number of words in the PFM set and the bandwidth of the power spectrum of a word.

Consider a PFM word and white additive Gaussian noise as an input to the bank of filters. The sinusoidal frequency of the word lies in the center of one of the N bandpass filters; and the probability density function for that filter will appear as in Figure 2(a), while those of the remaining N-1 filters will appear as in 2(b). Ultimately we are looking for a number which represents the ratio of the number of times that noise in one or more of the N-1 filters is greater than the signal-plus-noise in the correct filter. This figure is defined as the word-error probability.

In Figure 2(b), the shaded area represents the probability that the envelope amplitude of one filter is less than the threshold r_0. The probability that all the filters, excluding the one with the signal, have amplitudes less than r_0 is $P\left(r < r_0\right)^{N-1}$. The probability that at least one of the amplitudes is greater than r_0 then becomes $1 - \left\{P\left(r < r_0\right)\right\}^{N-1}$. By increasing the threshold r_0 by the amount Δr_0, we can set up the equality

$$1 - P\left(r < r_0\right) = 1 - \left\{P\left(r < r_0 + \Delta r_0\right)\right\}^{N-1} \tag{4}$$

which gives the same error rate for N-1 filters as that of a single filter. The fact that the signal amplitude would also be increased by an amount Δr_0 is more than compensated for by the fact that the information rate has been increased by $\log_2$ (100) over that of a single filter. If we wish to keep the information rate constant, the bandwidth of each of the N filters can be reduced by the factor $1/\log_2(100)$, substantially decreasing the signal power needed to maintain the same word-error probability at the same information rate.

The calculation of the word-error-probability curves for a set of 2^n uncorrelated filter outputs may proceed generally as above; i.e., the probability that an output is correct is calculated and then subtracted from unity to give the probability of error. The probability of one of the N-1 filters having an amplitude equal to or less than the envelope amplitude r_0 of the filter containing the signal is given by the following probability distribution function:

$$P\left(r < r_0\right) = \frac{1}{\sigma_x^2}\int_0^{r_0} r\, e^{-\frac{r^2}{2\sigma_x^2}}\, dr \tag{5}$$

However, the probability that no noise amplitude in any one of the N-1 filters exceeds r_0 is the probability distribution function raised to the power N-1. The probability that the signal-plus-noise in the correct filter has the amplitude r_0 is given by:

$$p\left(r_0\right) = \frac{r_0}{\sigma_x^2}\, e^{-\frac{r_0^2+C_0^2}{2\sigma_x^2}} \cdot \frac{1}{2\pi}\int_0^{2\pi} e^{\frac{r_0 C_0 \cos\left(\theta-\theta_0\right)}{\sigma_x^2}}\, d\theta\, . \tag{6}$$

Then the joint probability that none of the N-1 filters has noise exceeding r_0 and that the signal-plus-noise in the correct filter has the amplitude r_0 is:

$$p_c(N) = \left(\frac{1}{\sigma_x^2}\int_0^{r_0} r\, e^{-\frac{r^2}{2\sigma_x^2}}\, dr\right)^{N-1} \frac{r_0}{\sigma_x^2}\, e^{-\frac{r_0^2+C_0^2}{2\sigma_x^2}} \cdot \frac{1}{2\pi}\int_0^{2\pi} e^{\frac{r_0 C_0 \cos\left(\theta-\theta_0\right)}{\sigma_x^2}}\, d\theta\, dr_0 \tag{7}$$

To find the total probability that the correct filter amplitude is always greater than any of the amplitudes in the N-1 filters, the integration is performed over all values of r_0 from zero to infinity. The word-error probability $P_e(N)$ is then one minus this quantity:

$$P_e(N) = 1 - \int_0^{\infty} \left(\frac{1}{\sigma_x^2} \int_0^{r_0} r\, e^{-\frac{r^2}{2\sigma_x^2}} dr \right)^{N-1} \cdot \frac{r_0}{\sigma_x^2} e^{-\frac{r_0^2 + C_0^2}{2\sigma_x^2}} \cdot \frac{1}{2\pi} \int_0^{2\pi} e^{\frac{r_0 C_0 \cos(\theta - \theta_0)}{\sigma_x^2}} d\theta\, dr_0 . \quad (8)$$

This equation was programmed on an IBM 7090 digital computer for various numbers of filters, and the resulting curves are shown in Figure 3. The value of n in the figure is equivalent to $\log_2 N$ of Equation 15. This notation was maintained so that direct comparisons can be made to Viterbi's curves[10] for "greatest of" detectors using matched filters. There is, however, one difference to be noted. In the denominator of the abscissa of Figure 3 the noise power density is taken as kT_n watts/cps where k is Boltzmann's constant and T_n is the total noise temperature at the input to the detector. In the matched filter case Viterbi has defined the noise power density as $kT_n/2$ which assumes equal power at negative frequencies.

B. Multiple Matched Filter.

A matched filter performs essentially the process of cross-correlating the signal-plus-noise with a stored matching signal. It is actually autocorrelation of the signal with the stored image of the signal in the matched filter. The cross-correlation function

$$\frac{1}{T_0}\int_0^{T_0} f_l(t)\, f_m(t+\tau)\, dt ,$$

for $l = m$, is a maximum when $\tau = 0$. If other signals are added to the set such that

$$\frac{1}{T_0}\int_0^{T_0} f_l(t)\, f_m(t)\, dt = 0 \quad \text{for} \quad l \neq m , \tag{9}$$

then any one filter in the set of matching filters will respond to only one signal in the set; that is

$$\frac{1}{T_0}\int_0^{T_0} f_l(t)\, f_m(t)\, dt = 0 \quad \text{for} \quad l \neq m$$

and

$$\frac{1}{T_0}\int_0^{T_0} f_l(t)\, f_m(t)\, dt = C_{lm} \quad \text{for} \quad l = m .$$

By observing the output of the filters when $\tau = 0$ (maximum autocorrelation), a probability can be established that the matched filter with the greatest output contains the signal. The error probability is then one minus the probability of being correct.

We have not stated the waveform for the set of signals $f_l(t)$. It is possible to use any orthonormal set such as Legendre polynomials, Bessel functions, elliptic functions, and sine functions. PFM is based on the use of a set of sine functions having orthonormal properties.

In calculating the word-error probability for the matched filters, the same procedure is followed as for the case of unmatched filters except that the character of the noise is changed. This noise has a Gaussian probability density, and the probability that there is no amplitude greater than x_0 in a filter which has no signal is given by the integral of the Gaussian probability density function from $-\infty$ to x_0. Raising this probability distribution function to the power N-1 gives the probability that there is no output greater than x_0 in any of the N-1 filters. The probability that there is an output at x_0 in the correct filter and that none of the other filters has an amplitude greater than x_0 is

$$P_c(N) = \left(\frac{1}{\sqrt{2\pi}\,\sigma_m} \int_{-\infty}^{x_0} e^{-\frac{x^2}{2\sigma_m^2}} dx \right)^{N-1} \frac{1}{\sqrt{2\pi}\,\sigma_m} e^{-\frac{(x_0 - C_0)^2}{2\sigma_m^2}} dx_0 \,. \tag{10}$$

where $\sigma_m{}^2$ = variance of noise at output of matched filter, and C_0 = amplitude of signal at output of matched filter.

The probability of being correct for any amplitude x_0 is this integral over all values of x_0 from $x_0 = -\infty$ to $+\infty$; and the probability of an error is one minus that probability, or

$$P_e(N) = 1 - \int_{-\infty}^{\infty} \left(\frac{1}{\sqrt{2\pi}\,\sigma_m} \int_{-\infty}^{x_0} e^{-\frac{x^2}{2\sigma_{I_m}^2}} dx \right)^{N-1} \frac{1}{\sqrt{2\pi}\,\sigma_m} e^{-\frac{(x_0 - C_0)^2}{2\sigma_m^2}} dx_0 \tag{11}$$

These curves also have been programmed for computation on an IBM 7090 digital computer, and the results are plotted in Figure 4 for different values of n, the $\log_2$ of the number of filters used. The noise power density factor in the abscissa of Figure 4 is $kT_n/2$ as previously defined.

IV. EXPERIMENTAL RESULTS

A. Spectral Analysis

The operation of a set of contiguous filters combined with a maximum-likelihood detector can become quite complicated, especially when its signal is derived from magnetic-tape recordings of actual satellite passes. Since the output is influenced by many variables evaluation in conditions of poor signal-to-noise ratio is difficult. To obtain a more qualitative estimate of the value of the contiguous filter system, an experimental system was set up to make measurements under actual noisy conditions.

The equipment consisted of a tape scanner which performs the same task as an endless-loop tape recorder. The tape scanner has a seven-track reproducing head mounted on a revolving one-foot-diameter disk. Thus, one portion of a magnetic-tape recording can be scanned many times and the output observed on an oscilloscope.

By inserting a wave analyzer between the tape scanner and the oscilloscope, the spectral content of the signal for any pulse duration T and bandwidth $\Delta\omega_0$ can be obtained. A wave analyzer set at a bandwidth of 100 cps was used to obtain the envelope of the signal and noise over the range from 3.6 to 16.4 kc.

By very slowly changing the center frequency of the analyzer as a section of tape is scanned, a spectral analysis of the signal can be made. The output of the analyzer is used to intensity-modulate the beam of the oscilloscope as the sweep is being triggered at each revolution of the scanner. The analyzer's d. c. output voltage, which is proportional to the position of the frequency dial, is used on the y-axis of the oscilloscope in the same manner as the vertical deflection on a television raster. A time exposure of the oscilloscope face produces a high-quality spectral analysis of the signal: the x-axis represents time, the y-axis is a function of the frequency, and the intensity represents the energy in a filter at a particular time.

To demonstrate the noise-immunity characteristics of PFM, the signals from the satellite Explorer XII at a range of 65,000 km were processed with the experimental equipment. Two spectral-analysis photographs were taken of each telemetry frame; one covered the frequency range from 4.5 kc to 10 kc and the other from 10.0 kc to 18.0 kc. A composite photograph was made up from the twelve photographs of each six frames (Figure 5). To maintain high resolution along the frequency axis, the frequency dial was motor-driven at a rate sufficient to yield at least 300 horizontal lines per photograph; this gives an effective 600 lines in the vertical dimension.

This method of spectral analysis is not extremely effective for presenting noisy data, since the number of intensity levels that the human eye can distinguish is fairly limited. To increase the contrast between signal and noise, the wave analyzer output was passed through an RC network which added a small fraction of the derivative of the input to its output. The objective is to achieve a "three-dimensional" look by casting "shadows" on one side of each PFM pulse. Although very prominent on the original photographs, some of this effect is lost in reproduction.

V. CONCLUSIONS

A number of successful spacecraft have been flown with pulse-frequency modulation as the encoding technique. In decoding their analog data, a set of contiguous filters was used to extract the magnitude of the original signal. Since the individual filters were not flat within their passbands, degradation of the signal-to-noise ratio occurred if the encoded signal did not fall exactly in the center of a filter. Also, for response-time considerations, the filter bandwidth was equated to the reciprocal of the pulse length. By employing cross-correlation techniques using maximum-likelihood detectors, the measurement resolution can be doubled for the same pulse length without increasing the baseband width. This emphasizes an outstanding characteristic of pulse-frequency modulation: Quantizing errors for analog signals are not forced into the data at the source; rather, the quantization is accomplished during the data reduction. The manner of encoding the signals approaches the maximum communication efficiency; the amount of enhancement of the signal-to-noise ratio depends on the sophistication of the decoding process.

Error-probability curves have been derived for the unmatched-filter set with maximum-likelihood detection when perturbed by

Rayleigh noise. The matched-filter case has been given so that direct comparisons may be made. With large signal-to-noise ratios, the word-error probability as given by the Rayleigh probability density function approaches within 3 db the error probability for the matched filter given by the normal probability density function. At lower signal-to-noise ratios, the matched filter becomes appreciably better than the unmatched type; however, at these low values of signal-to-noise ratio the high error probability usually renders the data useless.

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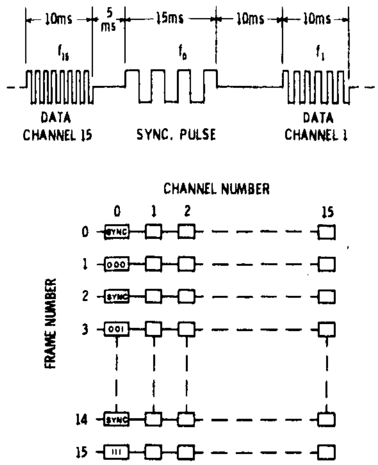

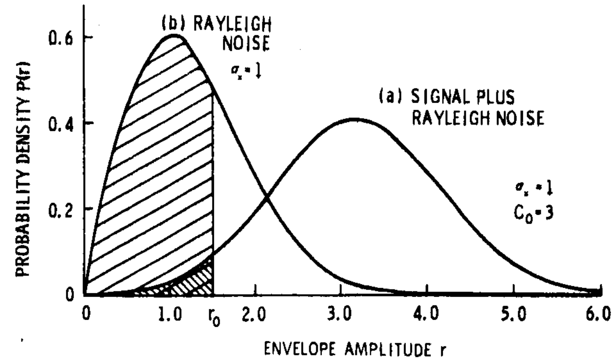

Figure 2. Probability Density Functions for (a) Signal Plus Rayleigh Noise, and (b) Rayleigh Noise.

Figure 1. Pulse-Frequency Modulation Format. Above, a telemetry frame. Below, a sequence format.

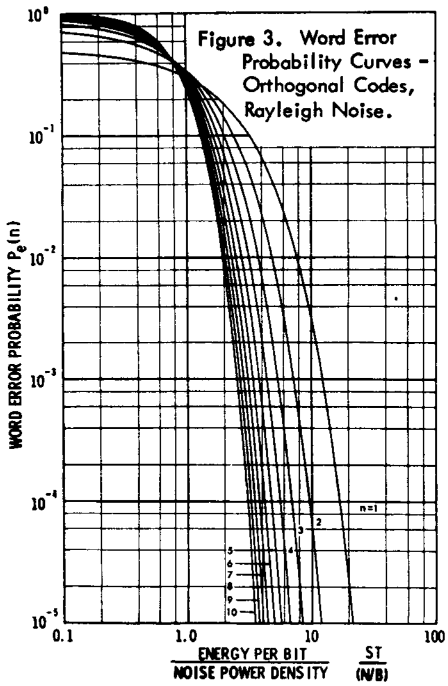

Figure 3. Word Error Probability Curves - Orthogonal Codes, Rayleigh Noise.

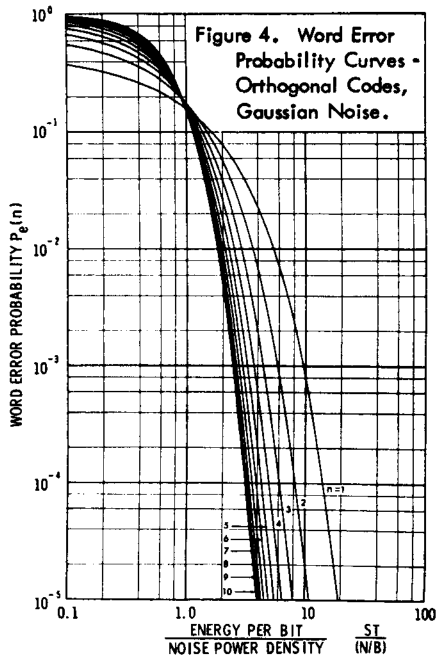

Figure 4. Word Error Probability Curves - Orthogonal Codes, Gaussian Noise.

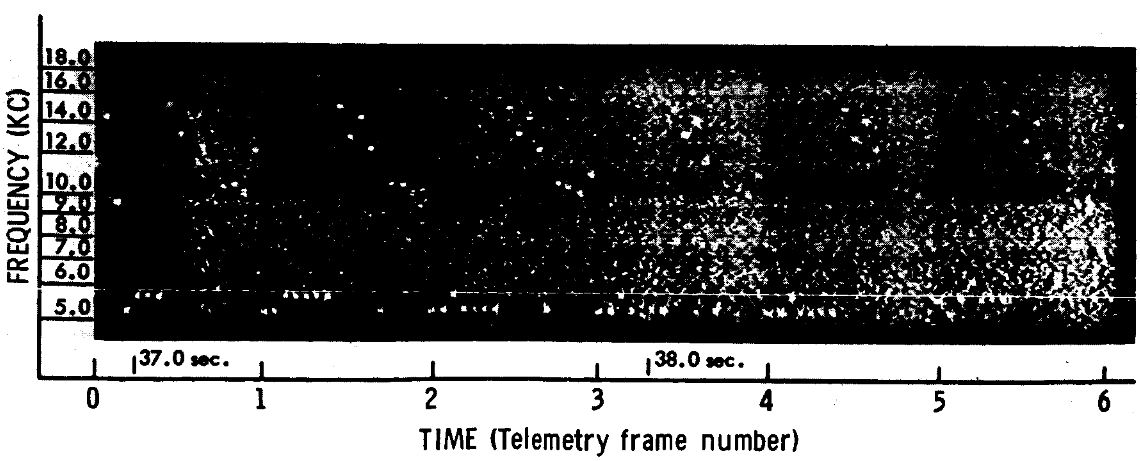

Figure 5. Spectral Analysis of PFM Telemetry Signals from Explorer XII, December 6, 1961, at 18:11. Range, 65,200 km; Power, 1.5 watts; Filter Bandwidth, 100 cps.